I. Fundamentals of Discrete Mathematics

1. Multiplication principle in combinatorics. Foundations of combinatorics: permutations, arrangements, combinations. Examples.
2. Inclusion-exclusion principle.
3. Combinations and their properties. Pascal's triangle. Newton's binomial theorem.
4. Partition of a finite set into subsets. Permutations with repetition. Polynomial formula.
5. Combinations with repetition and their properties.

II. Fundamentals of Analytic Geometry

1. Scalar (Dot) product of two vectors: definition, properties, component form of the scalar product.
2. Vector (Cross) product of two vectors: definition, geometrical interpretation, properties. The component form (the determinant form) of the vector product.
3. Scalar triple product of three vectors: definition, geometrical interpretation, properties. The component form (the determinant form) of the scalar triple product.
4. Lines in 2 dimensions. Equations of a line: normal vector form, parametric and Cartesian forms, two-point form, slope-intercept form, intercept form. Relationship between lines. The angle between two lines. Shortest distance from a point to a line.
5. Planes in 3 dimensions. Equations of a plane: normal vector form, Cartesian equation, three-point form, intercept form. Relationship between planes. Shortest distance from a point to a plane.
6. Lines in 3 dimensions. Equations of a line in 3-D. Relationship between lines in 3-D. Relationship between a line and a plane in 3-D, in particular the angle between a line and a plane and the point of intersection of a line and a plane.
7. Conics (second-degree curves): ellipse, hyperbola, parabola, their definitions, reduced canonical forms and optical properties.
8. Quadric surfaces: ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, cone, elliptic paraboloid, hyperbolic paraboloid, cylindrical surfaces. Reduced canonical equations of quadric surfaces.

III. Elements of Linear Algebra

1. Matrices: basic definitions, operations with matrices, applications.
2. Determinant of a matrix: definition, basic properties. Minors and cofactors, the Laplace's expansion theorem. The rank of a matrix.
3. Linear simultaneous equations. Cramer's rule. Theorem about principal (basic) minor. Consistency criterion for linear simultaneous equations (Kroneker – Capelli's theorem).

4. Homogeneous linear simultaneous equations: conditions for the existence of a non-trivial solution, fundamental set of solutions, the structure of the general solution. The form of the general solution to non-homogeneous linear simultaneous equations.
5. Linear vector spaces: definition, basic properties. Examples: the space R^n, the space of polynomials etc.
6. Linear operators: basic definitions. The space L(X,Y). Eigenvalues and eigenvectors of linear operators.
7. Linear, bilinear forms: the canonical form. Sylvester's law of inertia for the quadratic form.
8. Jordan normal form of a linear operator (matrix).
9. Functions of matrices and operators.

IV. Fundamentals of Mathematical Analyses

1. Number sequences. Limit of a sequence. Basic properties of convergent sequences. Lower and upper limits of a sequence. Limit of a monotone bounded sequence. The number *e*.
2. Limit of a function at a point. Basic properties.
3. Functions continuous at a point and on a segment. Basic properties.
4. Differential calculus: definition of a derivative of a function, geometrical interpretation of the derivative. Tangent line and normal to a curve. Rules of differentiation. Derivatives of higher orders. Differentials of the first and higher orders.
5. Antiderivative and Indefinite integral, their properties. Rules of integration.
6. Riemann integral. Necessary and sufficient conditions for Riemann integrability. Newton-Leibniz formula. Properties of definite integral.
7. Applications of definite integral to geometrical problems.
8. Vector functions of a scalar argument and their local properties.
9. Improper integrals of two kinds: integrals on infinite intervals; integrals with discontinuous integrand. Convergence and divergence of improper integrals. Absolute and conditional convergence of improper integrals. Abel-Dirichlet's test for convergence.
10. The Gamma and Beta functions, basic properties.
11. Functions of bounded variation. Jordan's theorem.
12. Riemann-Stieltjes integral, its properties and evaluation.
13. Numerical series: basic definitions. Convergent and divergent series, divergence test. Series with non-negative terms and tests for its convergence: comparison test, ratio test, root test, integral test.
14. Alternating series test (Leibnitz's theorem).
15. Functional series: point-wise and uniform convergence. Basic properties of uniformly convergent series.
16. Power series. Radius of convergence, interval of convergence. Abel's theorem. Cauchy- Hadamard theorem.
17. Taylor and Maclaurin series. Taylor series expansion for elementary functions.

18. Trigonometric Fourier series. Integral representation for partial sums of Fourier series. Point-wise convergence of Fourier series. Dini-Lipschits's test.
19. Uniform convergence of trigonometric Fourier series.
20. Fourier integral and Fourier transform, their basic properties.
21. Functions of several variables. Continuity of a function of several variables.
22. The idea of a directional derivative. The gradient. Partial derivatives.
23. Differentiability of a function of several variables: definition, necessary and sufficient condition for differentiability of a function of several variables.
24. Differential of a multivariable function.
25. Partial derivatives and differentials of higher orders. Tangent plane and normal to a surface.
26. Maximum and minimum values of functions of several variables.
27. Multiple integrals, their properties. Evaluation of multiple integrals.
28. Geometrical and mechanical applications of multiple integrals.
29. Line integrals of the first kind (curvilinear integrals): definition, properties, evaluation.
30. Line integrals of the second kind (line integral of a vector field): definition, properties, evaluation.
31. Scalar and vector fields. Flux and circulation of a vector field. Green's theorem. Gauss-Ostrogradsky theorem. Stoke's formula.
32. Potential vector field: definition, properties.

V. Fundamentals of Differential Equations

1. Ordinary Differential Equations (ODE) of the first order: basic definitions. Picard–Lindelöf theorem about existence and uniqueness of the solution to the Cauchy initial value problem.
2. Separable ODE's. Homogeneous first order ODE's.
3. Linear ODE's. General solution to a linear ODE. The method of variation of a constant.
4. Bernoulli ODE's. Complete solution and singular solution to the Bernoulli initial value problem.
5. Exact ODE's. Integrating factor, ways to find an integrating factor for an ODE.
6. Higher order ODE's that allow reducing of order.
7. Linear ODE's of higher orders – homogeneous and non-homogeneous ODE's. The fundamental set of solutions to a homogeneous linear ODE of n-th order and the structure of its general solution. The structure of the general solution to a non-homogeneous linear ODE of n-th order. The method of variation of constants.
8. Linear homogeneous ODE of n-th order with constant coefficients, its fundamental set of solutions and its general solution.
9. Linear non-homogeneous ODE of n-th order with constant coefficients. Method of undetermined coefficients.
10. Homogeneous linear simultaneous differential equations. Properties of their solutions. General solution to a system of linear ODE's.

VI. Fundamentals of Complex Analysis

1. The algebra of complex numbers. Operations with complex numbers. Geometrical representation of complex numbers (the Argand diagram). The modulus and the argument of a complex number. The extended complex plane. Stereographic projection and the Riemann sphere.

2. Functions of a complex variable. Regions in complex plane. The Jordan curve. The idea of a limit of a function of a complex variable. Continuity of a complex-valued function and basic properties of continuous functions. Uniform continuity theorem.

3. Complex power series. Convergence of a power series. First Abel's theorem. The disk and the radius of convergence of a power series. Uniform convergence of a power series. Second Abel's theorem.

4. Derivative of a function of a complex variable. Main properties and rules for differentiation. Cauchy-Riemann equations and their uses. Analytic functions and their properties. Definition of a singular point. Definition of a harmonic function. Interplay between a harmonic function and real/imaginary part of an analytic function.

5. Single-valued functions. Inverse functions. Elementary complex functions. Differentiation of a power series. Definition and basic properties of the complex exponent. Definitions and basic properties of complex trigonometric functions, their inverses. Definition and basic properties of the complex logarithm.

6. Complex integration. Integration of uniformly convergent series.

7. The Cauchy theorem. Antiderivatives and independence of path. Extension of the Cauchy theorem to multiple connected domains. Cauchy integral formula. Generalized Cauchy integral formula. The uses of Cauchy theorems.

8. The maximum modulus principle. Cauchy's inequality. Liouville's theorem and the fundamental theorem of algebra.

9. Taylor series. Series expansions for analytic functions.

10. Laurent series. The main and the principal part of the Laurent series. Zeros and singularities, examples. Classification of isolated singularities: removable singularities, poles of order n, in particular, simple poles, essential singularities. Behavior of an analytic function in a neighborhood of singularity.

11. Residue calculus: main definitions, computation of residues. Cauchy's residue theorem and its applications.

12. Uses of the residue theory. Argument principle. Rouche's theorem.

13. Fundamentals of the theory of conformal mappings. The principle of preservation of domains and the principle of an inverse mapping. Riemann-Schwarz symmetry principle.

14. The idea of a Laplace transforms. Definition and basic properties and theorems of Laplace transform. Region of convergence. Table of selected Laplace transforms.

15. Inverse Laplace transforms.

16. Laplace transforms method of solving initial value problems for ODE's.

17. Laplace transforms method of solving Volterra's integral equations.

VII. Basic Concepts of Probability Theory

1. Random events and operations with them.
2. Axiomatics of probability. Laws of probability.
3. Independent events. Multiplication principle. Conditional probability.
4. Law of total probability and Bayes' theorem.
5. Bernoulli trials. Binomial distribution.
6. Random variables. Probability distribution function (PDF) of a random variable, its properties. Examples.
7. Random vectors. Probability density function of a random vector, its properties.
8. Chebyshev's inequality and law of large numbers.
9. Dependence characteristics of random variables. Correlation coefficient: definition and properties.
10. De Moivre-Laplace theorem. Central limit theorem.

VIII. Fundamentals of Mathematical Physics

1. First order partial differential equations, the general solution, complete and singular integral. Geometrical approach to solving partial differential equations.
2. Second order partial differential equations. Classification of partial differential equations of the 2-d order and their canonical forms.
3. Classical partial differential equations – hyperbolic, parabolic and elliptic equations, classical problems for them.
4. Fourier method of separation of variables in the mixed problem for the heat equation.
5. The method of characteristics in a Cauchy problem for the vibrating string equation.

IX. Fundamentals of Measure Theory and Lebesgue Integration

1. Measures and their properties.
2. Lebesgue measure on a real line, on a plane and in R^n. Properties of the Lebesgue measure. Invariance of the Lebesgue measure under translation.
3. Measurable mappings and functions. Criterion for measurability of a function. Borel functions. Superposition of measurable mappings. Properties of measurable functions.
4. Simple functions and their properties. Measurability of simple functions. Theorem about any non-negative measurable function being a point-wise limit of simple functions.
5. Convergence in measure and its properties. Lebesgue's and Ries's theorems on almost everywhere convergence and convergence in measure.
6. Lebesgue integral: definition and properties.
7. Passing to the limit under the integral sign – Beppo-Levi theorem, Fatou' lemma, Lebesgue theorem on dominated convergence.

8. Lebesgue integral with respect to Lebesgue measure. Comparing Riemann and Lebesgue integrals on a segment. Comparing Riemann improper integrals and Lebesgue integrals on the real line.

X. Fundamentals of Functional Analysis

1. Metric spaces. Hölder's inequality and Minkowski's inequality.
2. Complete metric spaces. Examples. The nested sphere theorem. The Baire's category theorem. Banach contraction principle, fixed-point theorem and its applications.
3. Compact sets and their properties. Compactness criterion (the Hausdorff's theorem).
4. Compact sets in the space of continuous functions (the Arzela-Ascoli theorem).
5. Continuous functions on compact sets and their properties. The Stone-Weierstrass theorem.
6. Hilbert Spaces. Inner product in a Hilbert space. Euclidean Spaces. Orthogonal systems and bases. The orthogonalization theorem.
7. Bessel's inequality. Closed orthogonal systems. Parseval's identity. Complete Euclidean spaces, the Riesz-Fisher theorem.
8. The projection theorem in a Hilbert space and its applications. Orthogonal systems of functions in L_2-space.
9. Normed spaces. Banach spaces. Examples.
10. Linear operators, their properties. The norm of an operator.
11. Inverse and adjoint operators, their properties.
12. Linear operators in Hilbert Spaces. Hilbert-Schmidt operators.
13. The spectrum and the resolvent of a linear continuous operator. Compact operators and their properties.